*Int. J. Multiphase Flow* Vol. 16, No. 5, pp. 741–749, 1990
Printed in Great Britain. 
0301-9322/90 \$3.00 + 0.00


# PRESSURE WAVES OF MODERATE INTENSITY IN LIQUID WITH GAS BUBBLES

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(*Received* 6 *January* 1989; *in revised form* 5 *February* 1990)

**Abstract**—This paper reports the results of an experimental investigation into the evolution of pressure waves with intensity $1 < \Delta P_m/P_0 < 50$ in liquid gas bubbles of one or two different sizes.

*Key Words:* pressure wave, liquid, gas bubbles

## INTRODUCTION

To date, the propagation of small-amplitude pressure perturbations has been studied both theoretically and experimentally in works by Batchelor (1968), Noordzij & van Wijngaarden (1974) and Nakoryakov *et al.* (1983). These papers show that various types of perturbations—i.e. wave packets, oscillating shock waves, solitary waves (solitons)—exist in liquid with gas bubbles. The analysis and numerical calculations of stationary perturbations for waves of moderate intensity are presented in Goncharov *et al.* (1976). Experimental data on the structure of strong shock waves, reported by Deksnins (1978), show the presence of sharp pulsations with an amplitude which exceeds the average pressure in a shock wave. The existence of an oscillating solitary wave was found in liquid with gas bubbles of two different sizes by Gasenko *et al.* (1987). The properties of such complex wave structures, i.e. multisolitons, have been studied numerically by Gasenko & Izergin (1987).

The aim of this paper is to obtain experimental data on the dynamics and structure of pressure waves of moderate intensity in liquid with gas bubbles of one or two sizes over a wide range of wave and medium parameter variations, and also to study the behaviour of gas bubbles in a wave.

## EXPERIMENTAL SETUP

The experiments were conducted in a special "shock-tube" setup, a scheme of which is shown in figure 1. The test section (1), of length 1.5 m, i.d. = 53 mm and with a wall thickness of 8 mm, was made of a steel tube. It was filled with an aqueous–spirit–glycerine mixture and saturated with gas bubbles by means of two independent bubble generators (2). They were in the lower part of the test section. Calibrated capillaries, which are introduced directly into the test section and spaced uniformly along the length of the section, are applied in the design of bubble generators. In the large-bubble generator ($R = 1$–$2$ mm) steel needles with i.d. = 0.21 mm were used, but in the small-bubble generator ($R = 0.5$ mm) glass capillaries with i.d. = 0.05 mm were used. The void fraction was measured by variation of the liquid column height when introducing gas bubbles. Gases such as He, air, $CO_2$, freon-13 and freon-12 were used in the experiments. This allowed us to change the dissipative losses caused by heat transfer between the gas in a bubble and the surrounding liquid by an order of magnitude by changing the coefficient of gas temperature conductivity.

Initial pressure pulses were of a bell-like form and generated by the shock of a piston (3) against the moving bottom (4) of the transition chamber (5) filled with liquid. The piston is accelerated under the effect of compressed air upon rupture of the diaphragm (6) separating the high-(7) and low-(8) pressure chambers filled with air. The amplitude of the initial pressure pulse was varied by changing the pressure in the chamber (7). The profiles of the pressure waves were registered by piezoelectric pressure transducers with a sensitive element diameter $d = 2$ mm, which were placed

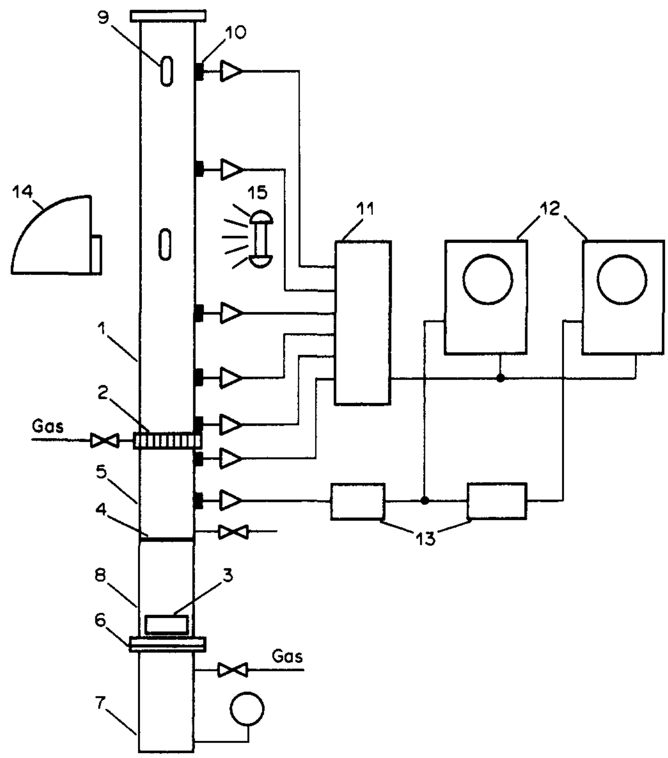



Figure 1. Principal scheme of the setup.

along the entire length of the working section. Signals from the pressure transducers were fed to the amplifiers (11) and then registered by electron-beam oscillographs (12). The character of the pressure variation with time at the transducer locations (10) was photographed from the oscillograph screens. The oscillograph scanning was initiated by the triggering sensor placed in the transition chamber, through the triggering generators (13).

The behaviour of gas bubbles in a pressure wave was studied while the wave was travelling using the camera, VSK-5 (14), at a rate of 50,000–150,000 frame/s. The moving pictures were taken in light transmitted from a flash lamp IFK-50 (15) synchronized with the camera shutter, through the optical windows (9) in the working section (1).

Figure 2 shows a histogram of the bubble scattering according to size for two characteristic bubble radii, $R_1 = 0.6$ and $R_2 = 1.2$ mm. One can see that deviation of the bubble size from the average value does not exceed 20%.

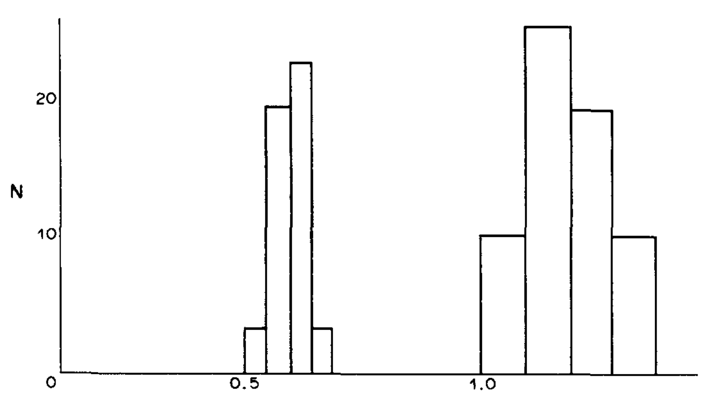



Figure 2. Histogram of bubble scattering according to size.

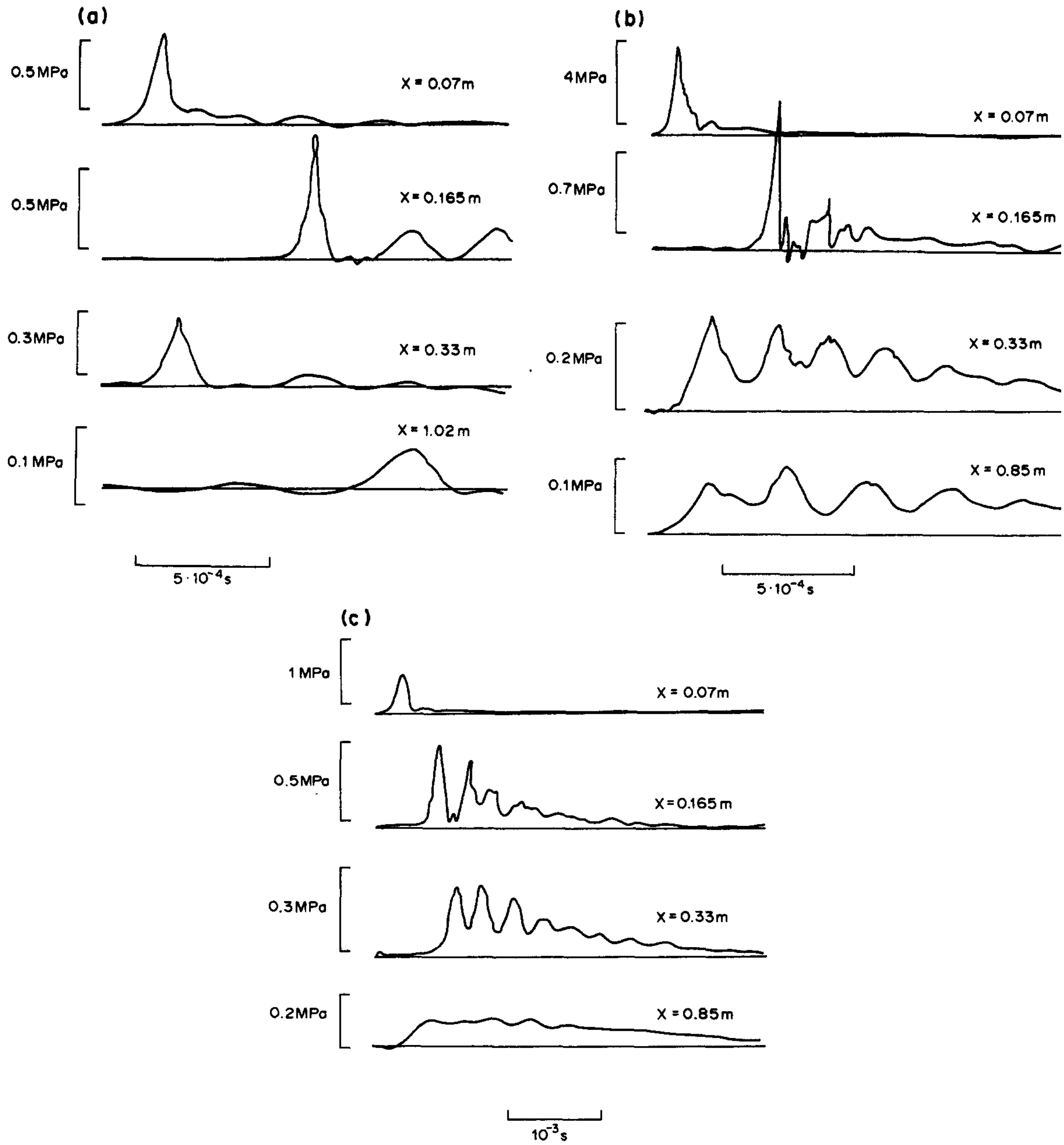



Figure 3. Evolution of pressure waves in liquid with gas bubbles of one size. $P_0 = 0.1$ MPa. (a) $CO_2$, $\epsilon_0 = 0.9 \cdot 10^{-2}$; (b) air, $\epsilon_0 = 0.8 \cdot 10^{-2}$; (c) He, $\epsilon_0 = 0.8 \cdot 10^{-2}$.

## EVOLUTION AND STRUCTURE OF PRESSURE WAVES IN LIQUID WITH GAS BUBBLES OF ONE SIZE

Figure 3 shows oscillograms of pressure wave profiles at distances $x$ from entry to the medium for various wave and medium parameters. One can see that a sequence of peaked solitary waves develops from the input bell-like signal. As the waves propagate, the amplitudes decrease but the duration increases. The formation of solitary waves occurs due to disperse effects in the bubble liquid which are related to the inertia of the attached gas inclusion mass. For small wave amplitudes the characteristic duration of perturbations is close to the resonance frequency of bubble oscillations. With an increase in the wave amplitude, the duration of solitary waves decreases due to strongly nonlinear bubble behaviour. The increase in the initial signal intensity and void fraction results in a stronger attenuation of the pressure waves.

Solitary waves did not interact with each other and propagated independently over the entire range of wave intensities $1 < \Delta P_m/P_0 < 50$ and void fractions $3 \cdot 10^{-3} \leqslant \epsilon_0 \leqslant 10^{-2}$ investigated in liquid with $CO_2$ bubbles of radius $R_0 = 1.2$ mm [figure 3(a)]. In the experiments with liquid containing air bubbles, attenuation of solitary waves was more intensive, and a qualitative change in the wave structure was observed for the initial signals of intensity $\Delta P_m/P_0 = 20$ [figure 3(b)]. The amplitude of the first solitary wave becomes comparable with that of subsequent waves and they are already coupled with each other at the initial stage of propagation due to strong attenuation. Coupled waves propagate as a single whole and attenuate considerably weaker than a solitary wave. Further, the wave amplitudes decrease and waves are transformed into an oscillating shock wave [figure 3(b)]. In the experiments the formation of an oscillating shock wave from the initial signal, whose pulsations were smoothed as the wave propagated, was observed in liquid with He bubbles over the entire range of wave amplitude investigated [figure 3(c)].

The investigation of wave attenuation is of great importance when studying the evolution of pressure waves in liquid with gas bubbles. A theoretical dependence for attenuation of solitons of small intensity $\Delta P_m/P_0 < 1$ in the presence of a slight dissipation was obtained by Pelinovsky (1971). This dependence is well-confirmed by the experimental results obtained by Kuznetsov *et al.* (1978). Their paper also shows that heat transfer between gas in the bubbles and the surrounding liquid is the basic mechanism of wave attenuation.

Figure 4 presents the experimental data on the attenuation of solitary waves of moderate intensity along the length $\Delta X = X_2 - X_1$ of the working section for various wave amplitudes $\Delta P$, void fractions $\epsilon$, bubble radii $R$ and temperature conductivities $a$ of gas in the bubbles. At small amplitudes $\Delta P_m/P_0 < 1$, wave attenuation is close to the theoretical dependence (line 4) obtained by Pelinovsky (1971). As the amplitude increases, wave attenuation increases more intensively than that calculated by Pelinovsky (1971). Criterial data processing has shown that data are generalized in dimensionless coordinates $\{\Delta P(X) = \Delta P_m(X_2)/\Delta P_m(X_1), H\}$ over the entire range of medium parameters investigated: $3 \cdot 10^{-3} \leqslant \epsilon_0 \leqslant 10^{-2}$, $5 \cdot 10^{-6} \leqslant a_0 \leqslant 2 \cdot 10^{-5}$ (m$^2$/C), $5 \cdot 10^{-4}$ (m) $\leqslant R_0 \leqslant 10^{-3}$ (m), wave amplitudes $1 \leqslant \Delta P_m/P_0 \leqslant 50$, where $H$ is of the form

$$H = \left[\frac{\Delta P_m(X_1)}{P_0}\right]^{0.67} \frac{\Delta X}{R_0}\left(\frac{a_0 \epsilon_0}{\omega_R R_0^2}\right)^{0.5}; \quad \omega_R = \left(\frac{3\gamma P_0}{\rho_0 R_0^2}\right)^{0.5}.$$

The dependence of wave attenuation on the medium parameters $\epsilon_0$ and $a_0$ obtained by Pelinovsky (1971), with due accounting for heat losses alone, has turned out to be preserved for higher intensities also. Generalization of the experimental data in the coordinates $\{P(X), H\}$ shows a predominant role of heat dissipation for solitary waves of moderate intensity.

The experimental data on pressure wave attenuation in liquid with He bubbles are not presented in figure 4. In this case in the experiments an oscillating shock wave whose attenuation differs from that of a solitary pressure wave forms immediately from the initial signal.

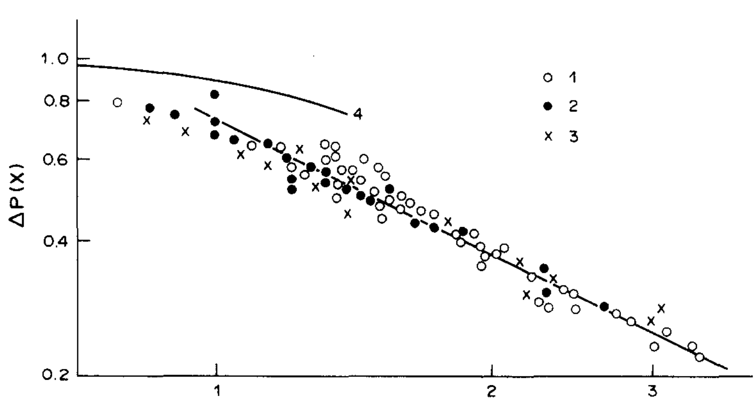



Figure 4. Attenuation of a solitary wave. 1—air; 2—$CO_2$; 3—freon-13; 4—calculation by Pelinovsky (1971)

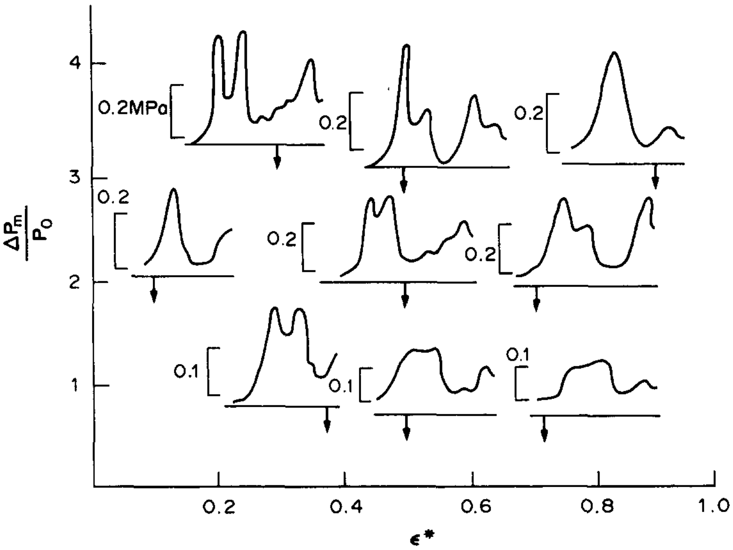



Figure 5. Map of wave structures. $R_1 = 0.6$ mm, $R_2 = 1.2$ mm, $P_0 = 0.1$ MPa.

## PRESSURE WAVES IN LIQUID WITH GAS BUBBLES OF TWO SIZES

A qualitative change in the pressure wave structure is observed upon pressure wave propagation in liquid with gas bubbles of two different sizes. As a result of the experiments performed, it has been shown that solitary waves with a characteristic oscillating profile exist in liquid with bubbles of two different sizes, and these waves have been investigated in detail.

Figure 5 presents a map of quasi-stationary wave structures, which generalizes the experimental data on pressure wave propagation in liquid with air bubbles of two sizes at the radius ratio $R_2/R_1 = 2$ depending on the wave amplitude $\Delta P_m/P_0$ and the dimensionless void fraction $\epsilon^* = (1 + \epsilon_1/\epsilon_2)^{-1}$; $\epsilon_1$ and $\epsilon_2$ are the void fractions of small and large bubbles, respectively, and $\epsilon_0 = \epsilon_1 + \epsilon_2$ is the total void fraction. For a small portion of large bubbles, $\epsilon^* \leqslant 0.1$, ordinary solitary waves, i.e. solitons, formed only by small bubbles are observed and no oscillating structures are found in the experiments. An oscillating solitary wave forms as the portion of large bubbles increases. In these cases large bubbles define the wave duration in the whole, and small ones define the oscillating period. The oscillating structure of a solitary wave is pronounced at $\epsilon^* \simeq 0.3$ and $\Delta P_m/P_0 \simeq 3$. A further increase in the portion of large bubbles results in decreasing the oscillation amplitude in a solitary wave, and at $\epsilon^* = 0.9$ ordinary solitons due to the oscillations of large bubbles alone are observed in the experiments. The existence of oscillatory solitary waves in such media is accounted for as follows. At pressure wave evolution a liquid with gas bubbles of two sizes can be presented as a system of two nonlinear oscillators. One of them is due to the presence of large bubbles and the other is due to the small bubbles. Nonlinear interaction between such oscillators is the reason for the formation of oscillating solitary waves. Numerical calculations by Gasenko & Izergin (1987) have shown that this results in the formation of stationary wave structures of complex form, i.e. multisolitons, at the propagation of nonlinear perturbations. Multisolitons with the oscillating mode (2, 1) are the simplest (figure 5). The mode $(n, m)$ means the number of oscillations of large $(m)$ and small $(n)$ bubbles in a wave.

Figure 6 presents the characteristic profiles of oscillating solitary waves with the mode (2, 1), which were observed by Gasenko & Izergin (1987) for two different void fractions of large bubbles. It is seen that oscillations of solitary waves decrease as the portion of large bubbles decreases, which corresponds to the numerical calculations of stationary wave structures. It has also been found experimentally that multisolitons with the mode (2, 1) form at wave amplitudes $\geqslant 3$ MPa, whereas the threshold of the formation of stationary multisolitons with the mode (2, 1) is 0.63 MPa in the numerical calculations by Gasenko & Izergin (1987). In addition, it has been obtained experimentally that ordinary solitons due to oscillations of large bubbles form at wave intensities $< 0.3$ MPa, but no regimes of wave stochastization, as predicted theoretically by Gasenko & Izergin (1987), are found in this case.

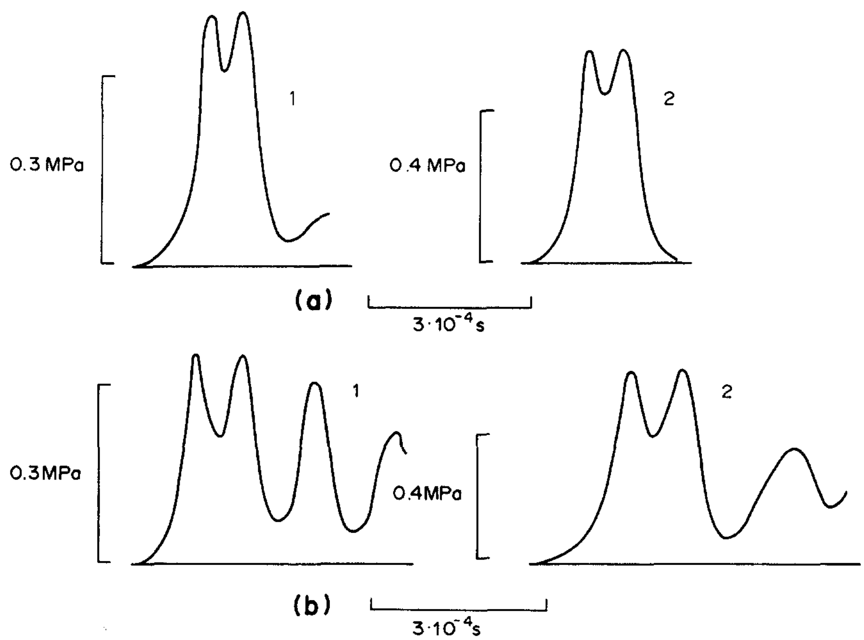



Figure 6. Structure of an oscillating solitary wave. $R_1 = 0.6$ mm, $R_2 = 1.2$ mm. (a) $\epsilon_1/\epsilon_0 = 0.3$; 1—experiment, 2—calculation by Gasenko & Izergin (1987). (b) $\epsilon_1/\epsilon_0 = 0.25$; 1—experiment, 2—calculation by Gasenko & Izergin (1987).

Figure 7 presents the characteristic oscillograms of pressure wave evolution in liquid with air bubbles at the radius ratio $R_2/R_1 = 3$. In this case a wave with the mode (3, 1) forms from the initial signal. The wave attenuates as it propagates, but the characteristic structure remains.

Complex oscillating structures changing their form as they propagate through a gas–liquid medium are observed in the experiments at the parameter values $1 < R_2/R_1 < 2$, which differ significantly from 1 and 2.

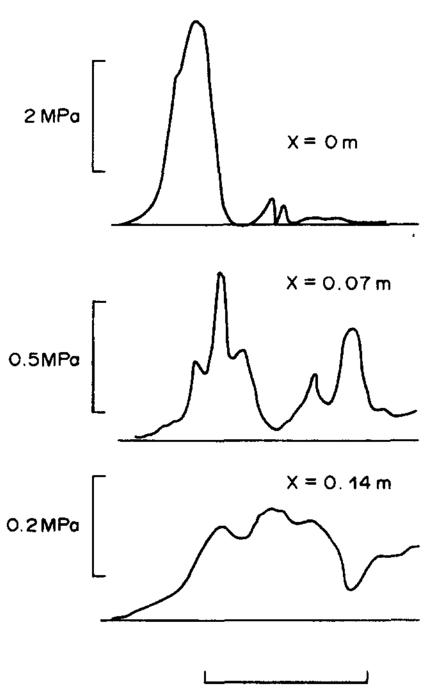



Figure 7. Evolution of pressure waves in liquid with air bubbles of two sizes. $\epsilon_0 = 0.01$, $R_1 = 0.6$ mm, $R_2 = 1.8$ mm, $\epsilon_1/\epsilon_0 = 0.6$.

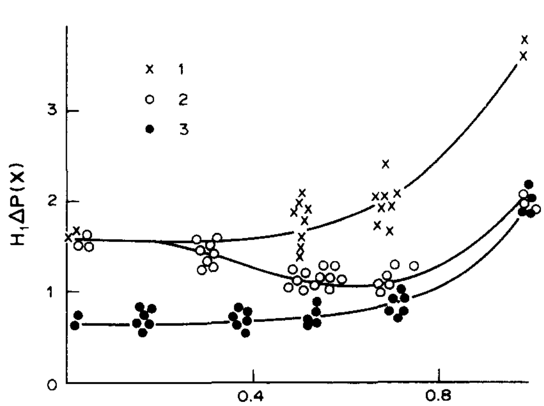
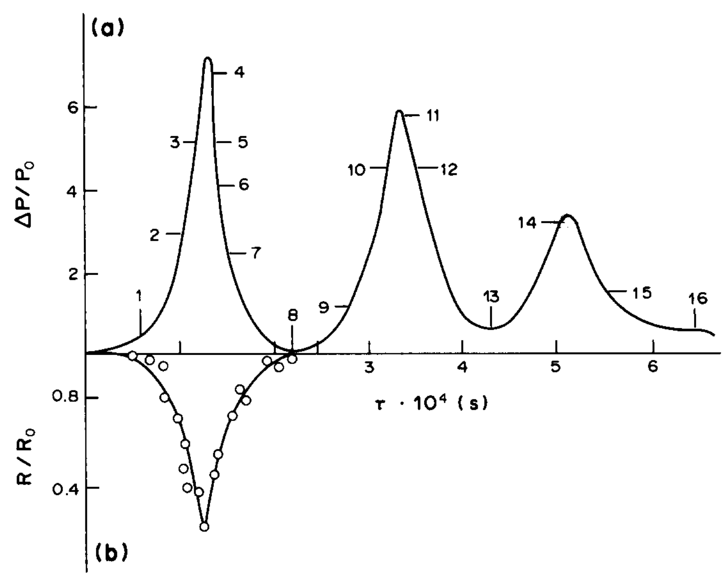



Figure 8. Attenuation of solitary waves of complex structure. Air: $R_1 = 0.6$ mm, 1—$R_2 = 1.2$ mm, 2—$R_2 = 1.8$ mm. 3—freon, $R_1 = 0.6$ mm, $R_2 = 1.2$ mm.

The parameter $H$ is used to compare the attenuation of oscillating solitary waves with that of ordinary solitons. Figure 8 presents the experimental data on solitary wave attenuation in liquid with gas bubbles of different sizes in the coordinates $H_1 \Delta P(X), \epsilon^*$, where $H_1 = H(R_1)$. One can see that when a small number of bubbles of small size ($R_1$) is introduced into a liquid with gas bubbles of a larger size ($R_2$) ($\epsilon^* \simeq 0.8$), wave attenuation increases sharply. This is attributed to the fact that the wave energy is mostly adsorbed by the small bubbles, whereas the pressure wave duration is determined by the oscillations of the large bubbles (figure 5), and the time of small bubble residence in a compressed state is maximal. Abnormal behaviour of wave attenuation in liquid with air bubbles (line 2) (i.e. decrease of attenuation on a further increase in the portion of large bubbles) is caused by the interaction of oscillating solitons due to strong heat dissipation, and the formation of an oscillating shock wave. Solitons do not interact in the experiments and the wave attenuation is of a monotonous character (line 3) for liquid containing bubbles of gas with a smaller coefficient of gas temperature conductivity (freon-13).

## BEHAVIOUR OF GAS BUBBLES IN A PRESSURE WAVE

The study of bubble behaviour in a solitary pressure wave of moderate intensity is of great interest from the viewpoint of constructing models of wave dynamics for gas–liquid systems.



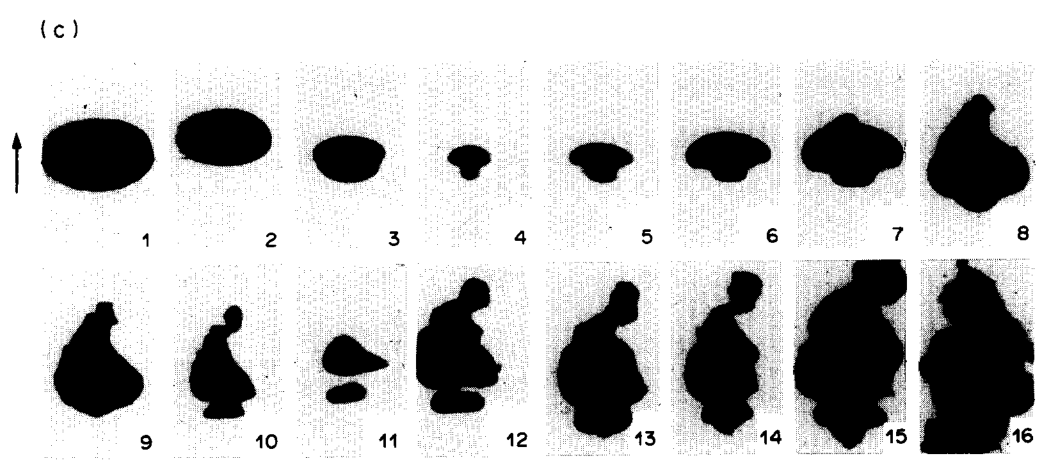




Figure 9. (a) Motion picture of the behaviour of a freon bubble in a pressure wave of moderate intensity. $P_0 = 0.1$ MPa, $\epsilon_0 = 8 \cdot 10^{-3}$, $R_0 = 1.2$ mm. (b) Variation of the average bubble size in a wave. (c) Motion pictures of the gas bubble behaviour at the moments in time denoted by the numerals on the pressure wave profile.

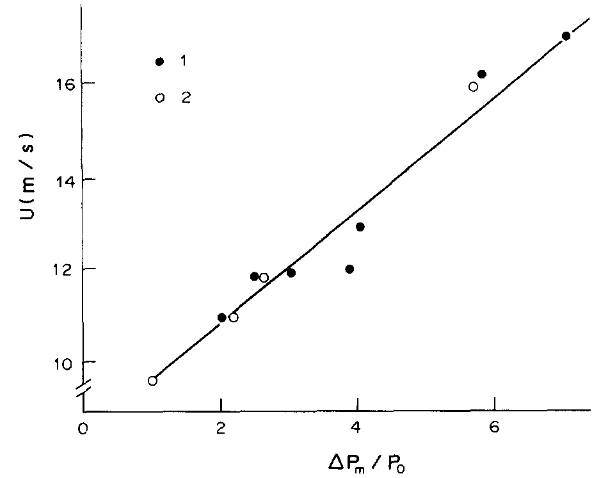



Figure 10. Dependence of exit velocity of the cumulative liquid jet from a bubble on wave intensity. 1—$R_1 = 1.2$ mm; 2—$R_1 = 0.6$ mm.

Figure 9 shows the oscillogram of the pressure wave profile in liquid with freon bubbles of one size at a distance $X = 0.33$ m from entry into the medium. One can see from the motion pictures that the bubble boundary loses its stability in the process of compression and the bubble is overcompressed in the direction normal to the wave propagation [the direction of wave propagation is denoted by the arrow in figure 9(c)]. This behaviour of the bubble surface seems to be accounted for by the formation of an annular liquid jet due to nonsphericity of the bubble and the presence of a pressure gradient on its surface. The analogous mechanism of annular jet formation was described by Voinov & Voinov (1976) while calculating the collapse of a nonspherical cavity near the solid wall. A cumulative liquid jet forms on the collapse of the annular jet. This jet is observed when it flows out of the back boundary of the bubble [figure 9(c)]. At subsequent oscillations of the bubble in the wave it is overcompressed again which results in its further destruction [figures 9(c) and 8]. While studying oscillations of a spherical bubble Kedrinsky & Soloukhin (1961) noted it was destroyed by the formation of a cumulative jet on the front boundary of the bubble. Thus, experiments have shown that in contrast to the results obtained by Kedrinsky & Soloukhin (1961) nonsphericity results in bubble fragmentation in the wave by the annular jet followed by the formation of a cumulative microjet.

Figure 10 shows the dependence of the penetration velocity $U$ of the microjet from the back boundary of the bubble into the liquid on the amplitude of the first pressure pulsation $\Delta P_m/P_0$. It appears that the penetration velocity does not depend on the bubble size and depends linearly

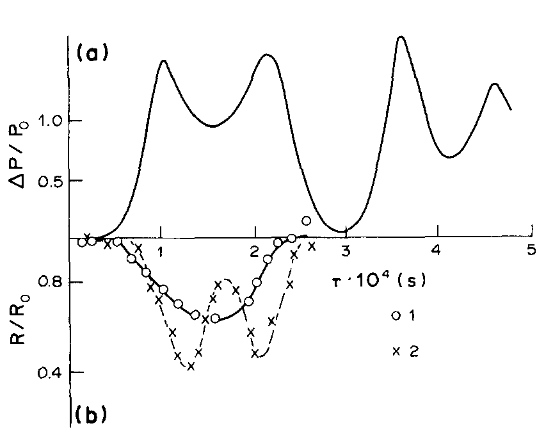



Figure 11. Variation of bubble radii in a pressure wave of complex structure. $P_0 = 0.1$ MPa, 1—$R_2 = 1.2$ mm, 2—$R_1 = 0.6$ mm. (a) variation of the bubble radii in a wave; (b) in the same time scale.

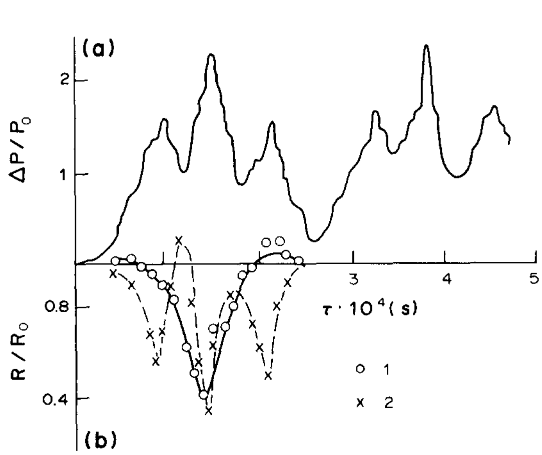



Figure 12. Variation of bubble radii in pressure wave of complex structure. 1—$R_2 = 1.8$ mm, 2—$R_1 = 0.6$ mm.

on the wave amplitude. At the same time, the velocity of the jet penetration from the bubble into the liquid is related to the velocity $v$ of the cumulative jet in the bubble and is equal to $v = 2U$ when liquid densities are equal in the jet and outside the bubble. [This formula was obtained by Lavrentyev & Shabat (1973).]

Figure 11 presents the pressure wave profile in liquid with freon bubbles with $R_2/R_1 = 2$. One can see that during the time of travel of the oscillating solitary wave the large bubble performs one oscillation and the small one performs two, in accordance with the model of two oscillators. This is confirmed by the resonance mechanism of multisoliton formation. In addition, it has been experimentally confirmed that a small bubble is in the compressed state for most of the time. Such small-bubble oscillations result in a more intensive heat transfer between the gas in the bubble and the surrounding liquid than has been found while investigating the attenuation of multisolitons with the mode (2, 1).

Figure 12 shows the structure of pressure waves in liquid with freon bubbles at $R_2/R_1 = 3$. In this case, one oscillation of a large bubble corresponds to three oscillations of a small bubble in the wave with the mode (3, 1), which also conforms with the resonance character of the formation of such a complex structure.

## REFERENCES


Batchelor, G. K. 1968 Compression waves in a suspension of gas bubbles in liquid. *Mechanik* **109,** 65–84.

Deksnins, B. K. 1978 Propagation of moderately strong shock waves in a two-phase medium. *Latv. PSR Zināt. Akad. Vest.* **1,** 75–81.

Gasenko, V. G. & Izergin, V. L. 1987 Nonlinear waves stochastization in a polydispersive gas–liquid mixture. In *Proc. 11th Int. Symp. on Nonlinear Acoustics*, Novosibirsk, part 2, pp. 23–26.

Gasenko, V. G., Dontsov, V. B., Kuznetsov, V. V. & Nakoryakov, V. E. 1987 Oscillating solitary waves in liquid with gas bubbles. *Izv. Akad. Nauk SSSR* **21,** 43–45.

Goncharov, V. V., Naugol'nykh, K. A. & Rybak, S. A. 1976 Stationary perturbations in liquid containing gas bubbles. *J. appl. Mech. tech. Phys.* **6,** 90–96.

Kedrinsky, V. E. & Soloukhin, R. I. 1961 Compression of spherical gas cavity in water by shock wave. *J. appl. Mech. tech. Phys.* **1,** 27–29.

Kuznetsov, V. V., Nakoryakov, V. E., Pokusaev, B. G. & Shreiber, I. R. 1978 Propagation of perturbations in a gas–liquid mixture. *J. Fluid Mech.* **85,** 85–96.

Lavrentyev, M. A. & Shabat, B. V. 1973 *Problems of Hydrodynamics and Their Mathematical Models.* Nauka, Moscow.

Nakoryakov, V. E., Pokusaev, B. G. & Schreiber, I. R. 1983 *Propagation of Waves in Gas– and Vapour–Liquid media.* ITF, Novosibirsk.

Noordzij, L. & van Wijngaarden, L. 1974 Relaxation effects caused by relative motion on shock waves in gas–bubbles fluid mixture. *J. Fluid Mech.* **66,** 115–143.

Pelinovsky, E. I. 1971 On absorption of nonlinear waves in dispersed media. *J. appl. Mech. tech. Phys.* **2,** 68–71.

Voinov, O. V. & Voinov, V. V. 1976 On the scheme of collapsing a cavitating bubble near the wall and formation of a cumulative jet. *Rep. Acad. Sci. USSR* **1,** 63–66.